Interplex Modulation

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In a conventional phase-shift-keyed/phase modulated (PSK/PM) system, the receiver tracks the frequency and phase of the carrier by means of a phase-locked loop and coherently demodulates the data. However, due to the inherent nonlinearity of the phase-modulation process, some power is transmitted as cross modulation, which reduces the useful available power. This article describes a new PSK/PM modulation scheme, called Interplex, which reduces the cross-modulation power loss. The scheme can be implemented in existing systems without significant hardware changes and appears attractive in concept for improving the performance of deep space telecommunications systems.

Introduction

In conventional multichannel frequency-multiplexed telemetry systems, several independent binary data channels are combined, using biphase-modulated subcarriers, to phase modulate a single RF carrier signal. Such systems are called phase-shift-keyed/phase-modulated (PSK/PM). The receiver tracks the frequency and phase of the carrier by means of a phase-locked loop and coherently demodulates the data. Lindsey (Reference 1) and Weber (Reference 2) analyzed the performance of such systems, which depends on the power in the data sidebands as well as on the unmodulated component of the carrier. However, due to the inherent nonlinearity of the phase-modulation process, some power is transmitted as cross modulation, which reduces the useful available power.

This article describes a new PSK/PM modulation scheme, called Interplex, which reduces the cross-modulation power loss, without affecting the basic structure of the conventional system. Thus, it can be easily incorporated into existing systems. It appears attractive in concept for improving the performance of deep space telecommunications systems.

Formulation

The general form of the phase-modulated RF signal in a multichannel PSK/PM system is

$$z(t) = \sqrt{2P} \sin [\omega_c t + \Theta(t)] \tag{1}$$

where P is the total average power, ω_c is the carrier frequency, and $\Theta(t)$ is the phase modulation. In conventional PSK/PM systems, $\Theta(t)$ is a linear combination of binary phase-shift-keyed sine-wave or square-wave subcarriers. We will temporarily assume that the system bandwidth is sufficiently large to handle square-wave subcarriers, because sine-wave subcarriers produce more cross-modulation loss and should be avoided. This of course may not be possible in very high data rate systems where bandwidth becomes a factor; however, the basic concepts are the same.

When a binary data stream $d_n(t) = \pm 1$ phase-shift-keys a unit amplitude square wave, sq $(\omega_n t) = \pm 1$, of frequency ω_n, the result is again a binary wave, $s_n(t) = \pm 1$,

$$s_n(t) = d_n(t) \text{ sq } (\omega_n t) \tag{2}$$

and the phase modulation $\Theta(t)$ in a conventional system is

$$\Theta(t) = \sum_{n=1}^{N} \theta_n s_n(t) \tag{3}$$

where N is the number of channels and the coefficients $\theta_1, \theta_2, \cdots, \theta_N$ are the modulation angles or modulation indexes whose choice determines the power allocation for each channel, the power allocated to the RF carrier, and the power that goes into cross modulation. We will analyze in detail the two-channel system. A generalization to the multichannel case will be briefly discussed in the last section.

Two-Channel Conventional PSK/PM System

When only two channels are involved, the phase-modulated RF signal is

$$\begin{aligned} z(t) = {} & [\sqrt{2P_1}\, s_1(t) + \sqrt{2P_2}\, s_2(t)] \cos (\omega_c t) \\ & + [\sqrt{2P_c} + \sqrt{2P_{cm}}\, s_1(t) s_2(t)] \sin (\omega_c t) \end{aligned} \tag{4}$$

where

$$\left.\begin{aligned} P_1 &= P \sin^2 \theta_1 \cos^2 \theta_2 = \text{power in channel 1} \\ P_2 &= P \cos^2 \theta_1 \sin^2 \theta_2 = \text{power in channel 2} \\ P_c &= P \cos^2 \theta_1 \cos^2 \theta_2 = \text{power in carrier} \\ P_{cm} &= P \sin^2 \theta_1 \sin^2 \theta_2 = \text{power in cross modulation} \end{aligned}\right\} \quad (5)$$

The cross modulation in the case of a two-channel system with square-wave subcarriers is simply the product of the two phase-shift-keyed subcarrier waveforms $s_1(t)s_2(t)$.

A block diagram of the receiver for this system is shown in Figure 1. The voltage-controlled oscillator (VCO) is in the phase-locked-loop arrangement that tracks the carrier component and produces the coherent reference signal cos $(\omega_c t + \phi)$, where ϕ is the receiver phase error. When $z(t)$ is multiplied by $\sqrt{2}$ cos $(\omega_c t + \phi)$ and the result is filtered to remove frequencies above ω_c, the result is a base-band signal of the form

$$\begin{aligned} y(t) = {} & [\sqrt{P_1}\, s_1(t) + \sqrt{P_2}\, s_2(t)] \cos \phi \\ & + [\sqrt{P_c} + \sqrt{P_{cm}}\, s_1(t)s_2(t)] \sin \phi + n_y(t) \end{aligned} \quad (6)$$

where $n_y(t)$ is the additive gaussian white noise.

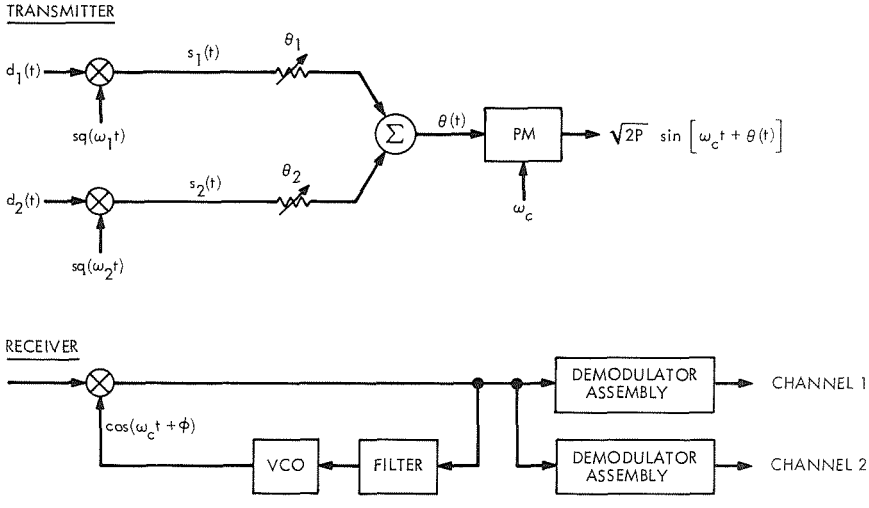

Figure 1. Conventional two-channel system

The data signals $d_1(t)$ and $d_2(t)$ are recovered from $y(t)$ by multiplying with locally generated coherent subcarriers sq $(\omega_1 t)$ and sq $(\omega_2 t)$ and filtering or bit detecting.

For a perfectly coherent system ($\phi = 0$), the efficiency of this system, measured in terms of the fraction of total power converted to data power, is

$$\eta = \frac{P_1 + P_2}{P} \tag{7}$$

From Equation 5 the total power is

$$P = P_1 + P_2 + P_c + P_{cm}$$

$$= P_1 + P_2 + P_c + \frac{P_1 P_2}{P_c} \tag{8}$$

since by Equation 5

$$P_{cm} = \frac{P_1 P_2}{P_c} \tag{9}$$

Since in the usual case, the requirement for data power in channel 1 results in $P_c < P_1$, it is clear that $P_{cm} \geq P_2$. Thus, the cross-modulation loss exceeds the power in the second (low rate) channel and indeed increases when P_c is decreased. Therefore, the total power needed is minimized when $P_{cm} = P_c = \sqrt{P_1 P_2} = P_c^*$ which corresponds to choosing $\theta_1 + \theta_2 = 90$ deg. Thus, the minimum total power is

$$P^* = P_1 + P_2 + 2\sqrt{P_1 P_2} = (\sqrt{P_1} + \sqrt{P_2})^2 \tag{10}$$

and the maximum efficiency is

$$\eta = 1 - \frac{2\sqrt{P_1 P_2}}{(\sqrt{P_1} + \sqrt{P_2})^2} \tag{11}$$

$$= 1 - \frac{2\sqrt{\alpha}}{(1 + \sqrt{\alpha})^2} \tag{12}$$

where $\alpha = P_2/P_1$ is the ratio of power in channel 2 to the power in channel 1. If both channels employ the same, or equally efficient, coding schemes, then α is also the ratio of data rates. However, in general, the lower rate channel could be more efficiently coded than the high rate link.

Referring to Equation 12 it is evident that the conventional PSK/PM two-channel system cannot be 100% efficient unless $\alpha = 0$, which corresponds to having a single channel ($P_2 = 0$). For two equal channels, i.e., $P_2 = P_1$ or $\alpha = 1$, $P_{cm} = P_c^* = P_2 = P_1$ and the efficiency is at most only

50%. It is worthwhile to observe that in this type of a system the carrier cannot be suppressed without suppressing one of the channels at the same time. Thus, power is wasted in two ways: one is in the transmission of unutilized cross-modulation power, which is larger than the low rate channel power, and the other is in the transmission of more than the required amount of unmodulated RF carrier power. These two sources of inefficiency are reduced by the Interplex modulation scheme.

Two-Channel Interplex

Let the phase-modulating signal be a linear combination of $s_1(t)$ and $s_1(t)\, s_2(t)$. The transmitted signal is

$$z(t) = \sqrt{2P} \sin [\omega_c t + \theta_1 s_1(t) + \theta_2 s_{cm}(t)] \tag{13}$$

where

$$s_{cm}(t) = s_1(t)\, s_2(t) \tag{14}$$

is the cross-modulation signal.

Again in Equation 4 the signal can be expanded trigonometrically into four terms; however, the second data channel will now be the cross-modulation term, while the new "cross modulation" will be

$$s_1(t)\, s_{cm}(t) = s_1^2(t)\, s_2(t) = s_2(t) \tag{15}$$

since $s_1^2(t) = (\pm 1)^2 = 1$.

The result is that second data channel and the cross-modulation term are interchanged, while the first data channel and the RF component remain unchanged. Therefore

$$\begin{aligned} z(t) = {} & [\sqrt{2P_1}\, s_1(t) + \sqrt{2P_{cm}}\, s_{cm}(t)] \cos(\omega_c t) \\ & + [\sqrt{2P_c} + \sqrt{2P_2}\, s_2(t)] \sin(\omega_c t) \end{aligned} \tag{16}$$

where now

$$\text{Interplex} \quad \left\{ \begin{array}{ll} P_1 = P \sin^2 \theta_1 \cos^2 \theta_2 = P_1 \\ P_2 = P \sin^2 \theta_1 \sin^2 \theta_2 = P_{cm} \\ P_c = P \cos^2 \theta_1 \cos^2 \theta_2 = P_c \\ P_{cm} = P \cos^2 \theta_1 \sin^2 \theta_2 = P_2 \end{array} \right\} \quad \text{Conventional} \tag{17}$$

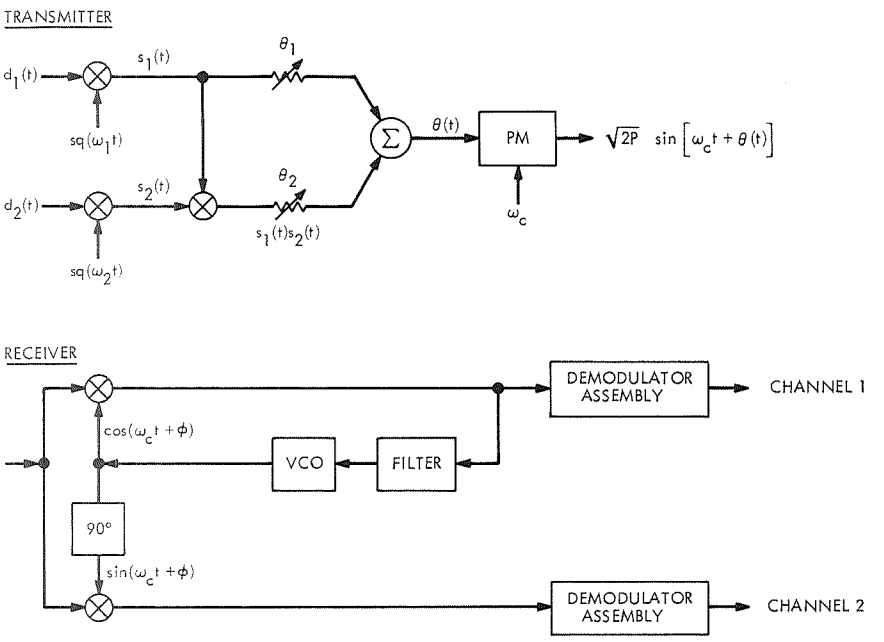

Figure 2. Interplex two-channel system

The only change relative to the conventional PSK/PM system is the use of a multiplier in the transmitter to form the product of the two binary signals $s_1(t)$ and $s_2(t)$. Since the result is also a binary signal, this requires only an *exclusive or* gate or a chopper, as shown in Figure 2. The receiver remains the same except that in addition to generating the coherent reference signal cos $(\omega_c t)$ its 90-deg phase shift sin $(\omega_c t)$ is also required. In this system $s_1(t)$ is obtained as in the conventional scheme, but $s_2(t)$ is obtained by mixing $z(t)$ with sin $(\omega_c t)$, as shown in Figure 2.

The power in the cross-modulation component

$$P_{cm} = \frac{P_c P_2}{P_1} = \alpha P_c \tag{18}$$

is now directly proportional to P_c and thus there is no advantage in transmitting more RF power than needed for tracking. Also, $P_{cm} \leq P_2$ while for the conventional scheme we had $P_{cm} \geq P_2$. In fact, of the four components, the cross modulation has the smallest power. The total power is

$$\begin{aligned} P &= P_1 + P_2 + P_c + \frac{P_2}{P_1} P_c \\ &= (1 + \alpha)(P_1 + P_c) \end{aligned} \tag{19}$$

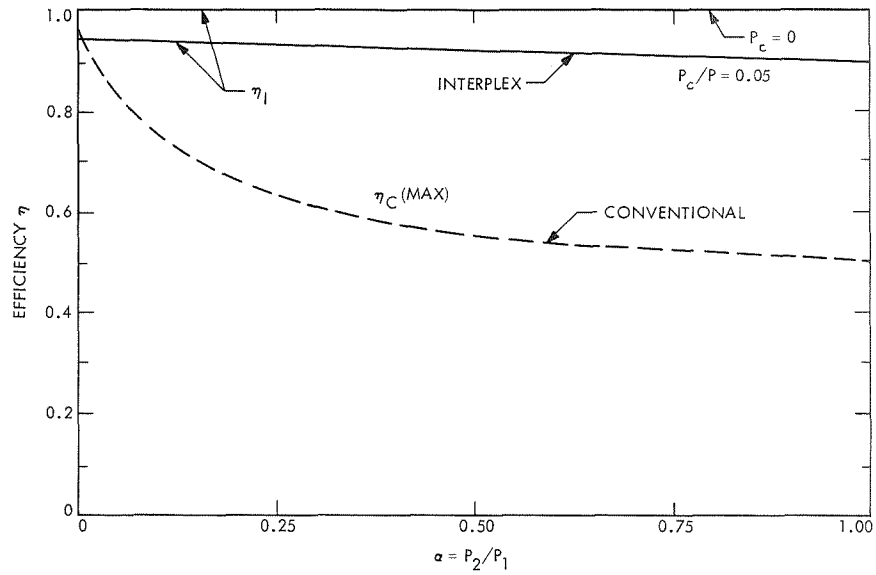

Figure 3. Data power efficiency of two-channel systems

where

$$\alpha = \frac{P_2}{P_1} = \tan^2 \theta_2 \tag{20}$$

Thus, the efficiency

$$\eta = 1 - \frac{P_c}{P_1 + P_c}$$

$$= \sin^2 \theta_1 \tag{21}$$

can be made large for all choices of α simply by taking P_c to be a small fraction of P_1, which is usually the case in practical systems. Moreover, it is now advantageous to completely suppress the carrier ($\theta_1 = 90$ deg) and achieve 100% efficiency for all ratios α. This can actually be realized because methods for tracking the phase of a suppressed carrier are theoretically feasible for the two-channel Interplex system (Reference 3).

A graph of the efficiency of the two-channel Interplex system and the conventional system as a function of α is given in Figure 3 for several values of P_c/P. Note that except for very small α the carrier power allocated by optimizing the conventional system is more than the amount required by the phase-locked receiver. The improvement gained by the Interplex scheme approaches 3 dB for the equal power case ($\alpha = 1$).

Multichannel Systems

In an N-channel PSK/PM system, the phase-modulated RF signal is still the same form as Equation 1, and the conventional modulation is given in Equation 3. However, as suggested by the two-channel Interplex system, we can let

$$\Theta(t) = \left[\theta_1 + \sum_{n=2}^{N} \theta_n \, s_n(t) \right] s_1(t) \tag{22}$$

Which of the two schemes is more efficient depends now on the power allocations required as well as on the number of channels. In general, Interplex is more efficient when N is not too large ($N \leq 4$ if the channels are equal, but larger values of N can be used when there is one high-power channel with many low-power channels). Two such cases are illustrated in Figure 4.

It is possible to show that no N-term linear combination of products of signals yields a better efficiency than either Interplex or conventional systems (Reference 4). However, if more than N terms are allowed (extended interplex) in a linear combination of signal products, then schemes are known (Reference 5) which outperform both the conventional and the simple interplex of Equation 22.

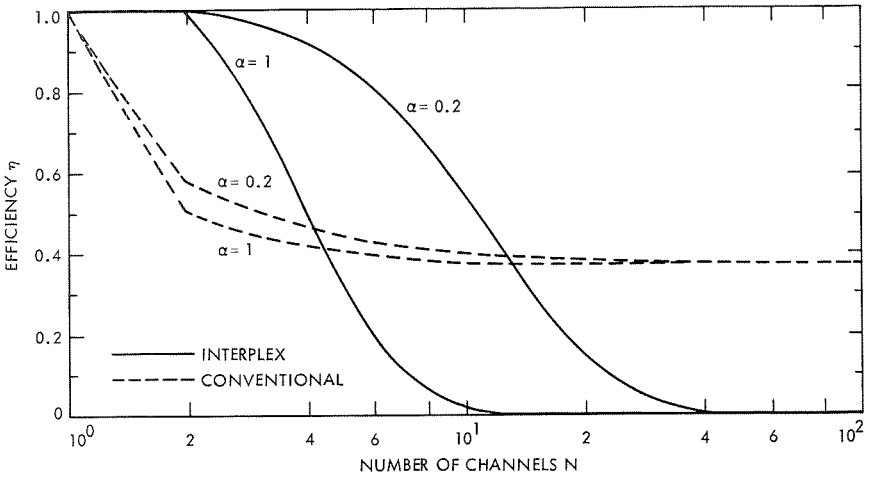

Figure 4. Efficiency for N channels with $P_k/P_1 = \alpha$, $k = 2, \cdots, N$

Conclusion

This article has presented a multichannel PSK/PM modulation scheme called Interplex which reduces cross-modulation loss and excess carrier reference power of conventional systems when the number of channels

is small. In systems with equal channels, Interplex is more efficient when the number of channels is four or less. The number of channels which can be combined efficiently via Interplex increases if there are many low-power channels and only one high-power link, as is often the case in deep space telemetry.

Although this article emphasized square-wave subcarriers, the basic ideas carry over when the subcarriers are sinusoids (Reference 6). The expressions for the various power terms will involve Bessel functions, and it is not difficult to show that Interplex offers significant improvement when N is small, as in the case of square-wave subcarriers. However, the performance with sinusoids is inferior to systems with binary waveforms.

Because of its promise of increased efficiency, further studies are being initiated to discover applicability to specific spacecraft missions. Experimental tests of the performance with simulated and actual ground and spacecraft telecommunications systems are also in progress.

References

1. Lindsey, W. C., "Design of Block-Coded Communication Systems," *IEEE Trans. Communication Technology*, Vol. COM-15, No. 4, Aug. 1967.

2. Weber, L. C., "Theory and Design of Coherent Digital Systems which Track Doppler Frequency," *Proc. IEEE*, Vol. 58, No. 6, June 1970.

3. Butman, S., and Timor, U., "Suppressed-Carrier Tracking for Two-Channel Phase Modulated Telemetry," *Proceedings of the National Electronics Conference*, Vol. 26, Dec. 1970.

4. Timor, U., "Optimum Configurations for PSK/PM Systems," in *Supporting Research and Advanced Development*, Space Programs Summary 37-66, Vol. III, pp. 33–36. Jet Propulsion Laboratory, Pasadena, Calif., Dec. 31, 1970.

5. Tausworthe, R. C., "A Boolean-Function Multiplexed Telemetry System," *IEEE Trans. Space Electronics Telemetry*, Vol. SET-9, No. 2, pp. 42–45, June 1963.

6. LaFrieda, J., "Space Station Unified Communication: Optimum Performance of Two-Channel High-Rate Interplex Systems," in *Supporting Research and Advanced Development*, Space Programs Summary 37-65, Vol. III, pp. 31–36. Jet Propulsion Laboratory, Pasadena, Calif., Oct. 31, 1970.